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Disturbing the random-energy landscape

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We examine the effects of correlated perturbations upon globally optimal paths through a random-energy landscape. Motivated by Zhang's early numerical investigations [Phys. Rev. Lett. **59**, 2125 (1987)] into ground-state instabilities of disordered systems, as well as the work of Shapir [Phys. Rev. Lett. **66**, 1473 (1991)] on random perturbations of roughened manifolds, we have studied the specific case of random bond interfaces unsettled by small random fields, confirming recent predictions for the instability exponents. Implications for disordered magnets and growing surfaces are discussed.

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I. INTRODUCTION

Much of the present interest in kinetic roughening phenomena [1] can be traced to the tremendous outpouring of research on the dynamic scaling properties of Eden clusters and ballistic deposits [2], following the introduction of a noisy Burgers equation by Kardar, Parisi, and Zhang (KPZ) [3]. In the past five years, we have discovered the extraordinary richness of this intriguing equation; within the realm of KPZ can be found a wonderfully varied collection of apparently unrelated physical problems, which, beyond the stochastic growth models mentioned above, include long-time tails of randomly stirred fluids, the asymptotics of flame-front propagation, as well as the roughening of vortex flux lines in ceramic superconductors. This last system [4], which in a different guise concerns the meandering of a directed polymer in a random medium (DPRM) [5,6], is essentially a small-scale version of the very difficult spin-glass problem [7] that has plagued the statistical physics community for the last decade, replete with an ultrametric free-energy landscape and potentially severe replica symmetry breaking [8], though blessed by a number of simplifying features that make it one of the few tractable problems of ill-condensed matter. Controlled by a strong-disorder zero-temperature fixed point, the DPRM is a classic global optimization problem in which one seeks to minimize the total energy of a directed path through a random environment. Performing averages of many realizations of the random-energy landscape yields highly nontrivial geometric and thermodynamic properties that characterize the ensemble of optimal paths. The present paper, motivated initially by the work of Zhang [9], and influenced by the later efforts of Shapir [10] and Mezard [11], investigates the resistance of these optimal paths to random disturbances of the disordered landscape. Our results address specifically the role of correlated drifts in the DPRM random-energy landscape and confirm important predictions for random-field (RF) perturbations of random-bond (RB) domain walls in disordered two-dimensional magnets.

II. MODEL

Our starting point is Zhang's formulation of the zero-temperature DPRM, which is most amenable to extensive numerical simulation. One considers a directed walker, who, starting at the origin of a square lattice, has the option of making an immediate step diagonally left or right to $(x,t)=(\pm 1,1)$. These and succeeding diagonal bonds have random energies drawn uniformly between 0 and 1. Neighboring bonds are uncorrelated. At the time slice t there are $t+1$ possible end points to the 2^t paths emanating from (0,0). As discussed earlier, the zero-temperature DPRM is simply a matter of global optimization, which, for a given realization of the random energy landscape (i.e., collection of random bonds on the lattice), entails finding the path of overall least energy, where the total energy of a path is given by the sum of the random bonds visited along the way. Many essential features of the $(1+1)$-dimensional DPRM were established early on; in particular, it is known that its geometric properties are controlled by transverse fluctuations off the central axis that scale as $x_{\rm rms} \sim t^{\zeta=2/3}$, while sample-to-sample fluctuations in the energy of the globally optimal trajectory scale as $e_{\rm rms} \sim t^{\omega=1/3}$, there being an index relation, $\omega=2\zeta-1$, connecting the energy and wandering exponents.

As stressed by Zhang [9], the globally optimal path through a given realization of the random-energy landscape is, however, quite susceptible to small changes in that random environment, there being many neighboring paths whose energies are very close to that of the ground state, but whose configurations differ considerably. These concerns have great physical import, of course, since true physical systems often possess quenched disorders that are actually dynamical variables, albeit on rather long time scales and with very small amplitudes. In his own numerical investigation into these issues, Zhang concentrated on the effects of an uncorrelated slow drift in the random energies of the bonds; that is, he considered adding to each random-bond energy an uncorrelated perturbation drawn with uniform probability between 0 and δ, with $\delta \ll 1$. In the context of disordered two-dimensional magnets, this corresponds to uncorrelated RB perturbations upon a RB interface. In the DPRM global optimization problem, this procedure yields two realizations of the random-energy landscape that are different, though produced from similar distributions and possessing substantial overlap. Because of the work of Shapir [10], one knows that there exists a crossover length scale $t^* \sim \delta^{-1/\varphi}$, where $\varphi_{\rm RB}=\frac{1}{6}$ for random-bond perturbations and $\varphi_{\rm RF}=\frac{1}{2}$ for random-field perturbations, beyond which the small differences between the perturbed and unperturbed realizations of the random-energy landscape manifest themselves in an asymptotic fashion. There are, however, a number of interesting scaling properties associated with ground-state instabilities of this disordered system that reveal themselves immediately. We focus our attention upon them first.

III. NUMERICAL DATA

Consider for example, the fact that the two globally optimal paths in the two different but highly correlated random environments are typically quite distinct. If x_1, x_2 denote the transverse positions of these two best paths, then we find for RF's perturbing RB interfaces that the mean jump scales with path length as, see Fig. 1, $|x_1 - x_2| \sim t^{\alpha=1.16\pm0.02}$, entirely consistent with Shapir's prediction that

$$\alpha_{\rm RF}=\varphi_{\rm RF}+\zeta_{\rm RB}=\tfrac{1}{2}+\tfrac{2}{3}=\tfrac{7}{6}\ .$$

Our own data for the case of RB's perturbing RB's, which Zhang considered in his original work and are presented here for the sake of comparison, are shown in Fig. 2, corroborating the mean jump exponent $\alpha_{\rm RB}=\varphi_{\rm RB}+\zeta_{\rm RB}=\frac{1}{6}+\frac{2}{3}=\frac{5}{6}$. In both instances, we have used $\delta=0.1$ and performed disorder averages over 4000 realizations of the random-energy landscape. Furthermore, it is apparent that in the case of RF's perturbing RB's, where the crossover length scale $t^*_{\rm RF} \sim \delta^{-2} \sim 100$, the data begin to pull away from the straight-line fit. By contrast, for RB's perturbing RB's, the data follow the line well beyond accessible system sizes since $t^*_{\rm RB} \sim \delta^{-6} \sim 10^6$ steps in this case.

Note that this mean jump scaling index is quite large because of the ultrametric structure of the ensemble of locally optimal paths. The very best path in a given realization of randomness is stable with respect to its immediate family thanks to the substantial ancestry they have in common. Hence, there is an intrinsic resistance to change, which Zhang alludes to as a Hopfield memory effect. Nevertheless, because of the random perturbations upon the original disordered landscape, a distant relative of the original best path can accumulate enough energy gains to become the globally optimal trajectory in the perturbed landscape, incurring a large transverse jump in the process. Since the triumphant neighbor is rarely a local relative, these jumps make important contributions to the statistical averages.

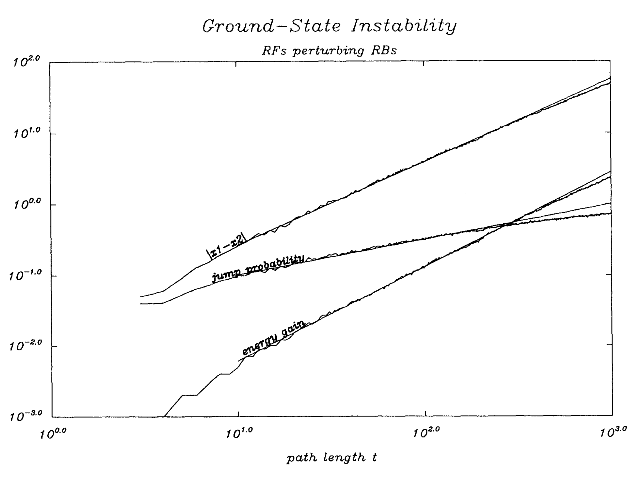

FIG. 1. RF perturbations of the DPRM. From the left: Top curve, mean jump distance; middle curve, jump probability, bottom curve, energy advantage of new best path over old best path.

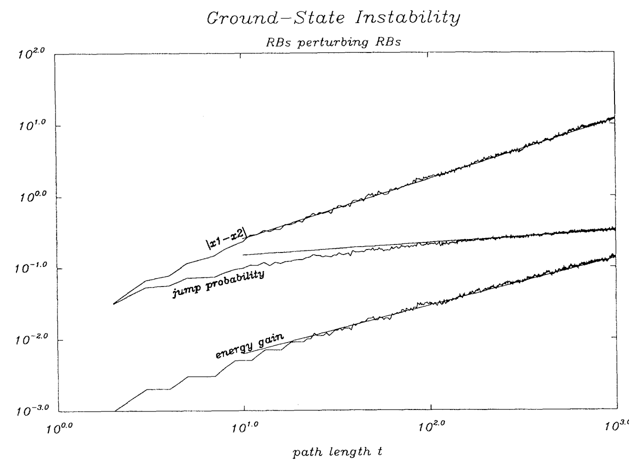

FIG. 2. Same as previous figure, but for RB perturbations of the DPRM—the case considered by Zhang.

In addition to the mean jump size, we have also studied the probability that a jump actually happens $(x_1 \neq x_2)$ for the case of RF perturbations of the random-energy landscape. A glance at Fig. 1 reveals that for paths shorter than the crossover length scale, the data are well fit by a line of slope $\frac{1}{2}$. For RB's perturbing RB's, the data fall along a line of slope $\frac{1}{6}$, as explained by Feigel'man and Vinokur [12]. Again, for the RB case, there is no indication, whatsoever, of incipient crossover phenomena. Note that, while there are geometric arguments [12–14] that predict these jump probability exponents, for both RF's and RB's, they coincide with the crossover index φ.

All this is easily understood on the basis of Shapir's scaling ansatz for the interfacial roughness $|x(t)|$,

$$|x_\delta(t)|=|x_0(t)|g(\delta t^\varphi)=|x_0(t)|\{1+g'(0)\delta t^\varphi+\cdots\}\ ,$$

from which it follows that the displacement in the perturbed environment is given by

$$\Delta x(t)=|x_\delta(t)|-|x_0(t)| \sim |x_0(t)|g'(0)\delta t^\varphi \sim \delta t^{\varphi+\zeta}\ ,$$

whence the mean jump exponent, while

$$P(\text{jump}) \sim \Delta x(t)/|x_0(t)| \sim \delta t^\varphi\ .$$

For reasons that are unclear, the energy-based derivation of Feigel'man and Vinokur [12] for the jump probability exponent appears not to carry through for RF perturbations of the RB landscape. Nonetheless, the linear dependence of both the mean jump distance and the jump probability on the strength of the perturbation were manifest in the RF simulations we performed for other values of δ [15]. Zhang had pointed out similar behavior in the RB case.

Given the quickly growing jump probability it is natural to wonder whether the old best path in the original random-energy landscape retains some honor by remaining a locally optimal path in the new disordered environment. In our numerical studies we found that the energy change of the old best path in the different energy environments had an exponent of unity for RF's, which suggests that the new locally optimal path does indeed overlap substantially with the old best path. Finally, we investigated the energy advantage, in the new environment, that motivated the jump away from the old optimal path. See Figs. 1 and 2. It scales with an exponent $\omega'_{\mathrm{RF}}=\frac{4}{3}$ for RF perturbations, $\omega'_{\mathrm{RB}}=\frac{2}{3}$ for RB perturbations. The latter index had been noted, of course, by Zhang. Nevertheless, our RF simulation provides additional support to the conjecture that the instability exponents obey a scaling relation, $\omega'=2\alpha-1$, analogous to that of the unperturbed problem.

Finally, in Fig. 3, we illustrate the long-term implications of RF perturbations upon the RB landscape. Whereas the data for geometric and free-energy fluctuations in the original unperturbed random-energy landscape scale nicely and are consistent with the exponents $\zeta_{\mathrm{RB}}=\frac{2}{3}$ and $\omega_{\mathrm{RB}}=\frac{1}{3}$, in the new environment RF perturbations incur crossover to the stronger fluctuations characteristic of correlated roughening, the exponents $\zeta_{\mathrm{RF}}=\omega_{\mathrm{RF}}=1$ in agreement with those predicted by Imry-Ma–type arguments [16] for the two-dimensional (2D) RF Ising model. Note that, while deviations manifest themselves first for the sample-to-sample fluctuations of the energy, the march to asymptotic scaling is well underway for both quantities once the length scale $t^*_{\mathrm{RF}}\sim 100$ is crossed.

It is clear from our numerical studies that correlated random perturbations can incur severe ground-state instabilities in uncorrelated disordered systems. In the case of the DPRM, a RF perturbation upon the RB landscape can have increasingly drastic consequences for the configuration of globally optimal paths, causing large jumps to distant relatives beyond the immediate family. Ultimately, of course, the scaling is controlled entirely by the RF fixed point, characterized by exponents quite different from those of the RB problem. These effects are presumably observable in 2D RB magnets that are subject to a very weak external magnetic field, giving rise to small perturbing RF's within the sample [17]. Thermally activated jumps of domain walls to minimal energy configurations might manifest themselves as large detectable noises in measurements of the magnetization, susceptibility, etc. For the kinetic roughening of stochastically grown surfaces, the importance of a spatially correlated perturbation in the atomic beam would be dramatic, leading to radically different surface morphologies with substantially different scaling properties.

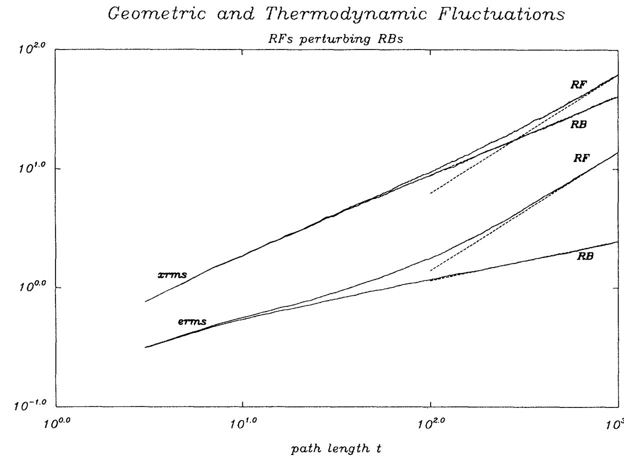

FIG. 3. *Position* (top pair) and *energy* (bottom pair) fluctuations of the globally optimal path. Upper curves within each pair correspond to the best path in the disordered environment perturbed by random fields. For RF's perturbing RB's, both the position and energy fluctuations eventually scale with unit slope, in agreement with Imry-Ma predictions for the 2D RF Ising model. Lower curves, associated with the original unperturbed energy landscape, exhibit standard 1+1 DPRM exponents, $\zeta_{\mathrm{RB}}=\frac{2}{3}$, $\omega_{\mathrm{RB}}=\frac{1}{3}$.

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